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Tips, tricks and formulae on H.C.F and L.C.M

Highest Common Factor (H.C.F)

- H.C.F is the highest common factor or also known as greatest common divisor, the greatest number which exactly divides all the given numbers.

  There are two methods to find H.C.F of given numbers, they are:

  i. Prime factorization method.
  ii. Division Method.

  How to find H.C.F of given numbers by prime factorization method?

  Follow the steps below to find H.C.F of given numbers by prime factorization method.

  1. Express the given numbers as product of their prime factors
  2. Check for the common prime factors and find least index of each common prime factor in the given numbers
  3. The product of all common prime factors with the respective least indices is H.C.F of given numbers

  Ex:
  H.C.F of 12, 36, 48
  1. Express the numbers as product of prime factors

  $12 = 3 * 2^2$

  $36 = 3^2 * 2^2$

  $48 = 3 * 2^4$

  2. The common prime factors are 2 and 3 and the corresponding least indices are 2 and 1 respectively

  3. The product of all the common prime factors with the respective least indices

  H.C.F of 12, 36, 48 = $2^2 * 3$ = 12

  How to find H.C.F of two numbers by division method?

  Follow the steps below to find H.C.F of two numbers by division method

  1. Divide higher number with the smaller number
  2. Divide smaller number with the remainder of above division
  3. Then second remainder divides first remainder and the process of division continues till remainder is zero.

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4. Last divisor of division is the H.C.F of two numbers

Ex:

H.C.F of 12 and 56

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12)56
   48
    8)12
       8
       4)8
         8
         0
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The last divisor in the above division is 4.

Hence, H.C.F of 12 and 56 = 4

Least Common Multiple (L.C.M)

- L.C.M is least common multiple, the smallest number which is exactly divisible by all the given numbers.

  There are two methods to find L.C.M of given numbers, they are:

  i. Prime factorization method.
  ii. Division Method.

How to find L.C.M of given numbers by prime factorization method?

Follow the steps below to find L.C.M of given numbers by prime factorization method.

1. Express the given numbers as product of their prime factors.
2. Find highest index in all the prime factors of given numbers.
3. The product of all the prime factors with respective highest indices is the L.C.M of given numbers.

Ex:

L.C.M of 14, 42, 36

1. Express the numbers as product of prime factors.

$$14 = 2 * 7$$

$$36 = 3^2 * 2^2$$

$$42 = 2 * 3 * 7$$

2. The highest index of 2, 3, 7 are 2, 2, 1 respectively

3. The product of all the prime factors with the respective highest indices.

L.C.M of 14, 36, 42 = $2^2 * 3^2 * 7$ = 252

How to find L.C.M of given numbers by division method?

Follow the steps below to find L.C.M of given numbers by division method.

1. Divide all the numbers with the common prime factors.
2. The division process continues until there is no common factor to the numbers.
3. The product of divisors and remaining numbers with no common factor is the L.C.M of given numbers.

Ex: L.C.M of 12, 98, 188

$$\begin{array}{r|l} 2 & 12,98,188 \\ \hline 2 & 6,49,94 \\ \hline & 3,49,47 \\ \hline \end{array}$$

The product of divisors and remaining numbers = 2*2*3*49*47 = 27636

Hence, the L.C.M of 12, 98, 188 = 27636

HCF of given fractions

- H.C.F of given fractions = $\frac{\text{H.C.F of numerators}}{\text{L.C.M of denominators}}$

Ex:

Find the H.C.F of the fractions $\frac{4}{9}, \frac{16}{15}, \frac{12}{21}$

According to the formula, H.C.F of numerators 4, 16, 12

$$4 = 2^2$$
$$16 = 2^4$$
$$12 = 2^2 * 3$$

So, the H.C.F of numerators = 2^2 = 4

L.C.M of denominators 9, 15, 21

$9 = 3^2$

$15 = 3*5$

$21 = 3*7$

So, the L.C.M of denominators = 3^2*5*7 = 315

Therefore, H.C.F of given fractions = $\frac{\text{H.C.F of numerators}}{\text{L.C.M of denominators}} = \frac{4}{315}$

L.C.M of given fractions

- L.C.M of given fractions = $\frac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}}$

  Ex:

  Find the L.C.M of the fractions $\frac{2}{3}, \frac{7}{18}, \frac{11}{12}$

  According to the formula, L.C.M of numerators 2, 7, 11

  As the numerators are all prime numbers, L.C.M of numerators = 2*7*11 = 154

  H.C.F of denominators 3, 18, 12

  $3 = 3^1$

  $18 = 2*3^2$

  $12 = 2^2*3$

  So, H.C.F of denominators 3, 18, 12 = 3

  Therefore, L.C.M of given fractions = $\frac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}} = \frac{154}{3}$

How to compare fractions by using L.C.M of denominators

- Comparison of fractions
  1. Find L.C.M of the denominators in given fractions
  2. Find the resultant fraction for all the numbers with above L.C.M as denominator
  3. Arrange the corresponding fractions in the order of their numerators of resultant fractions

Ex: Arrange the fractions in descending order $\frac{7}{3},\frac{12}{5},\frac{1}{9}$

The L.C.M of denominators 3, 5, 9 = $3^2 * 5 = 45$

Convert each fraction with the L.C.M as equivalent denominator,

$\frac{7}{3} = \frac{x}{45} \Rightarrow x = 105$, fraction is $\frac{105}{45}$

$\frac{12}{5} = \frac{y}{45} \Rightarrow y = 108$, fraction is $\frac{108}{45}$

$\frac{1}{9} = \frac{y}{45} \Rightarrow y = 5$, fraction is $\frac{5}{45}$

$\frac{108}{45} > \frac{105}{45} > \frac{5}{45}$ so the corresponding fractions are $\frac{12}{5} > \frac{7}{3} > \frac{1}{9}$

Things to remember

- The H.C.F of two or more numbers is smaller than or equal to the smallest number of given numbers.
- The L.C.M of two or more numbers is greater than or equal to the greatest number of given numbers.
- The smallest number which is exactly divisible by x, y and z is L.C.M of x, y, z.
- If the H.C.F of the numbers a, b, c is K, then a, b, c can be written as multiples of K (Kx, Ky, Kz, where x, y, z are some numbers). K divides the numbers a, b, c, so the given numbers can be written as the multiples of K.

- If the H.C.F of the numbers a, b is K, then the numbers (a + b), (a -b) is also divisible by K.
  The numbers a and b can be written as the multiples of K, a = Kx, b = Ky.
  (a + b) = (Kx + Ky) = K (x + y)
  (a - b) = (Kx - Ky) = K(x – y)
  Therefore, (a + b) and (a - b) is also divisible by the H.C.F of a, b.
- Product of two numbers = H.C.F *L.C.M of the two numbers (**NOTE:** applicable only for two numbers)
  Ex: The two given numbers are 198, 68

  Express 198, 68 as product of their prime factors

$198 = 2 * 3^2 * 11$

$68 = 2^2 * 17$

H.C.F of the two numbers = 2

L.C.M of 198, 68 = $2^2 * 3^2 * 11 * 17$ = 6732

Product of two numbers = 198*68 = 13464

H.C.F * L.C.M = 6732*2 = 13464

Hence, product of two numbers = HCF * LCM of the two numbers.

- Two numbers are said to be co-prime if their H.C.F is 1.

Formulae

1. H.C.F of given fractions = $\frac{\text{H.C.F of numerators}}{\text{L.C.M of denominators}}$
2. L.C.M of given fractions = $\frac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}}$
3. Product of two numbers = H.C.F *L.C.M of the two numbers (**NOTE:** applicable only for two numbers)

Model questions:

- The smallest number which when divided by x, y and z leaves a remainder R in each case.
  Required number = (L.C.M of x, y, z) + R

- The greatest number which divides x, y and z to leave the remainder R is H.C.F of (x – R), (y – R) and (z – R)
  Ex:
  Find the greatest number divides 24, 60, and 84 leaves the remainder 3

  By definition the greatest number which divides the given numbers and leaves a remainder of 0, only if it is the H.C.F of given numbers

  x = 24, y = 60, z = 84 and R = 3

H.C.F of (x - R), (y - R), (z - R) = H.C.F of 21, 57, 81

$21 = 3 * 7$

$57 = 3 * 19$

$81 = 3^4$

So, H.C.F of 21, 57, 81 =3
Therefore the greatest number is 3 which divide 24, 60 and 84 to leave a remainder 3

- The greatest number which divide x, y, z to leave remainders a, b, c is H.C.F of (x - a), (y - b) and (z - c)
  Ex:
  Find the greatest number which divides 18, 26 and 54 to leave remainders 3, 1, 4
  Here, x = 18, y = 26, z = 54, a = 2, b = 1, c = 4

  H.C.F of (x - a), (y - b), (z - c) = H.C.F of 15, 25, 50

  $15 = 5 * 3$

  $25 = 5^2$

  $50 = 2 * 5^2$

  So, H.C.F of 15, 25, 50 = 5, we can cross check by dividing x, y, z and obtain the same remainders as mentioned in question.

- The smallest number which when divided by x, y and z leaves remainder of a, b, c
  (x – a), (y - b), (z - c) are multiples of K

  Required number = (L.C.M of x, y and z) – K

  Ex:

  Find the Smallest number which divides 2, 5, 7to leave remainders 0, 3, 5
  Here, x = 2, y = 5, z = 7, a = 0, b = 3, c = 5
  (x - a), (y - b), (z - c) = 2, 2, 2

  Therefore, K = 2

  L.C.M of x, y, and z = 2 * 5 * 7 = 70

  Required number = (L.C.M of x, y and z) – K = 70 -2 =68

  The number 68 can be cross verified by dividing 2, 5 and 7 which leaves remainder0, 3, and 5 as mentioned in the question

Points to remember

1. The smallest number which when divided by x, y and z leaves a remainder R in each case.
   Required number = (L.C.M of x, y, z) + R

2. The greatest number which divides x, y and z to leave the remainder R is H.C.F of (x – R), (y – R) and (z – R)

3. The greatest number which divide x, y, z to leave remainders a, b, c is H.C.F of (x - a), (y - b) and (z - c)

4. The smallest number which when divided by x, y and z leaves remainder of a, b, c
   (x – a), (y - b), (z - c) are multiples of K
   Required number = (L.C.M of x, y and z) – K