Tips, tricks and formulae on Boats & streams

Definition & Formulae

- **Downstream movement**: When the direction of a boat or swimmer is in the same direction of stream, then it is called as downstream movement.

  If, Speed of the stream = S
  Speed of the boat or swimmer in still water = B,

  The boat or swimmer is moving in same direction of the stream, so this helps the boat move faster than its speed in still water. Therefore, the relative speed of boat or swimmer is the sum of speed of boat and speed of river.

  Therefore, Downstream Speed of boat or swimmer = (B + S)

  Ex:

  Find the distance travelled by the boat in direction of stream for 3hours, speed of the boat in still water is 5 km/h and speed of the stream is 2km/h.

  Speed of the boat or swimmer in still water = 5 km/h

  Speed of the stream = 2 km/h

  Time taken = 3 hours

  Downstream Speed of the boat = (B + S) = (5 + 2) km/h = 7 km/h

  Distance travelled = Downstream Speed * Time taken = 7 * 3 km = 21 km. $\left(\because Distance = Speed * Time\right)$

- **Upstream movement** : When the direction of a boat or swimmer is in opposite direction of stream, then it is called as upstream movement.

  If, Speed of the stream = S
  Speed of boat or swimmer in still water = B,

  The boat or swimmer is moving against the direction of stream, so the speed of boat is reduced than its speed in still water. Therefore, the relative speed of boat or swimmer is subtraction of speed of the stream from speed of boat.

  Therefore, Upstream Speed of boat or swimmer = (B - S)
  ($\because$ The speed of the boat or swimmer is considered more than the speed of river else the boat or swimmer would not be able to move against the stream)

Ex:

Find the time taken by a man to swim 6 km against the direction of stream. Speed of the man in still water is 6 km/h and speed of the stream is 3 km/h.

Speed of the man in still water = 6 km/h

Speed of the stream = 3 km/h

Distance to swim = 6 km

Upstream Speed of the man = (B - S) = (6 - 3) km/h = 3 km/h

Time taken = $\frac{\text{Distance}}{\text{Upstream Speed}} = \frac{6}{3} = 2 hours \left(\because Time = \frac{Distance}{Speed} \right)$

- When upstream and downstream speed of boat or swimmer given.

1. Speed of boat or swimmer in still water = $\frac{1}{2}$ (Downstream speed + Upstream speed)
2. Speed of the stream = $\frac{1}{2}$(Downstream speed – Upstream speed)

As mentioned in the previous two points, the downstream speed and upstream speed of boat or swimmer is (B + S) and (B - S) respectively.

Speed of boat or swimmer in still water = $\frac{1}{2}$(Downstream speed + Upstream speed) = $\frac{1}{2}$ (B + S + B - S) = B

Speed of river = $\frac{1}{2}$(Downstream speed - Upstream speed) = $\frac{1}{2}$((B + S) – (B - S)) = $\frac{1}{2}$(B + S – B + S) = S

Formulae:

Speed of boat or swimmer in still water = B,

Speed of the stream = S,

1. Downstream Speed of boat or swimmer = (B + S)
2. Upstream Speed of boat or swimmer = (B - S)
3. Speed of boat or swimmer in still water = $\frac{1}{2}$ (Downstream speed + Upstream speed)
4. Speed of the stream = $\frac{1}{2}$(Downstream speed – Upstream speed)

Model questions:

- The speed of a boat in still water is B km/h and the speed of stream is S km/h. The time taken by the boat to reach a certain place and come back to its initial position is T.
  1. Find distance between the two places.
  2. Find average speed of the boat.

  Sol:

  Speed of the boat in still water = B km/h

  Speed of the stream = S km/h

  In this situation, the boat travels along the stream and also against the stream,
  Downstream speed of the boat = (B + S) km/h
  Upstream speed of the boat = (B - S) km/h

  Let the time taken by the boat to reach the place is T1, assume that it is downstream movement.

  Let the time taken by the boat to return back to the initial position from that place is T2, assume that it is upstream movement.

  Let distance between two places be D.

  So, Downstream speed = $\frac{\text{Distance travelled}}{\text{Time taken}} \Rightarrow (B+S) = \frac{D}{T1} \Rightarrow T1 = \frac{D}{(B+S)}$

  Upstream speed = $\frac{\text{Distance travelled}}{\text{Time taken}} \Rightarrow (B-S) = \frac{D}{T2} \Rightarrow T2 = \frac{D}{(B-S)}$

  The total time taken to reach back to initial position = T = T1 + T2

  Therefore, T = $T = T1 + T2 = \frac{D}{(B+S)} + \frac{D}{(B-S)}$

$$\Rightarrow D\left(\frac{1}{(B+S)}+\frac{1}{(B-s)}\right)=D\left(\frac{B-S+B+S}{(B+S)(B-s)}\right)=D\left(\frac{2B}{\left(B^2-S^2\right)}\right)$$

$$\Rightarrow T=D\left(\frac{2B}{\left(B^2-S^2\right)}\right)\Rightarrow D=\frac{T\left(B^2-S^2\right)}{2B}$$

Average speed of the boat = $\frac{\text{Total distance travelled}}{\text{Total time taken}}=\frac{D}{T}=\left(\frac{\frac{T\left(B^2-S^2\right)}{2B}}{\frac{T}{1}}\right)=\left(\frac{\left(B^2-S^2\right)}{2B}\right)$

1. Distance between the two places is $D=\frac{T\left(B^2-S^2\right)}{2B}$
2. Average speed of the boat = $\left(\frac{\left(B^2-S^2\right)}{2B}\right)$

- The speed of a swimmer in still water is B km/h and the speed of stream is S km/h. The time taken by the swimmer to swim same distance in downstream is T1 and in upstream is T2. Find the ratio of speed of a swimmer to the speed of stream.

  Sol:
  Speed of swimmer in still water = B km/h
  Speed of stream = S km/h
  Downstream speed of the swimmer = (B + S) km/h
  Upstream speed of the swimmer = (B - S) km/h
  Let the distance = D,

  Downstream speed = $\frac{\text{Distance travelled}}{\text{Time taken}}=\frac{D}{T1}\Rightarrow(B+S)=\frac{D}{T1}$

  Upstream speed = $\frac{\text{Distance travelled}}{\text{Time taken}}=\frac{D}{T2}\Rightarrow(B-S)=\frac{D}{T2}$

Therefore,

$$\Rightarrow D=(B+S)T1, D=(B-S)T2$$
$$\Rightarrow (B+S)T1=(B-S)T2$$
$$\Rightarrow BT1+ST1=BT2-ST2$$
$$\Rightarrow ST1+ST2=BT2-BT1$$
$$\Rightarrow S(T1+T2)=B(T2-T1)$$
$$\Rightarrow \frac{B}{S}=\frac{(T1+T2)}{(T2-T1)}$$

The ratio of speed of the swimmer to the speed of stream $=\frac{B}{S}=\frac{(T1+T2)}{(T2-T1)}$

Points to remember:

Speed of boat or swimmer in still water = B,

Speed of the stream = S,

1. If the total time taken by the boat to row a distance of D and reach back to its initial position is T

   Then,

   I. Distance between the two places is $D=\frac{T\left(B^2-S^2\right)}{2B}$

   II. Average speed of the boat $=\left(\frac{\left(B^2-S^2\right)}{2B}\right)$

2. If the time taken by the boat to row same distance in downstream is T1 and in upstream is T2.

   Then, the ratio of speed of the boat to the speed of stream $=\frac{B}{S}=\frac{(T1+T2)}{(T2-T1)}$